The System of Multi Color-flux-tubes in the Dual Ginzburg-Landau Theory

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Abstract

We study the system of multi color-flux-tubes in terms of the dual Ginzburg -Landau theory. We consider two ideal cases, where the directions of all the color-flux-tubes are the same in one case and alternative in the other case for neighboring flux-tubes. We formulate the system of multi color-flux-tubes by regarding it as the system of two color-flux-tubes penetrating through a two dimensional sphere surface. We find the multi flux-tube configuration becomes uniform above some critical flux-tube number density $\rho_c = 1.3 \sim 1.7\mathrm{fm}^{-2}$. On the other hand, the inhomogeneity on the color electric distribution appears when the flux-tube density is smaller than ρ_c. We discuss the relation between the inhomogeneity in the color-electric distribution and the flux-tube number density in the multi-flux-tube system created during the QGP formation process in the ultra-relativistic heavy-ion collision.

I. INTRODUCTION

The QCD vacuum is non-trivial at zero temperature. In this vacuum, quarks and gluons are confined in hadrons and the chiral symmetry is spontaneously broken. As the temperature increases, however, the color degrees of freedom in hadrons are defrozen. Above the critical temperature, the QCD vacuum is in the quark-gluon plasma (QGP) phase, where quarks and gluons move almost freely. The lattice QCD simulation demonstrates that such a phase transition happens at about 280 MeV for the pure gauge case [1] and at about 100$\sim$200MeV for the full QCD case [2].

In the recent years, some experimental groups are trying to create QGP as the new form of matter in the laboratory using high-energy heavy-ion collisions. The RHIC (Relativistic Heavy Ion Collider) project is aimed at forming QGP and at studying its properties. The scenario of producing QGP is based on Bjorken's picture [3]. Just after heavy ions pass through each other, many color-flux-tubes are produced between the projectile and the target, and pulled by them as shown in Fig.1(a). Usually, it is guessed that these flux-tubes are cut into several pieces through quark-antiquark pair creations, and these short flux-tubes, which behave as excited 'mesons', are thermalized by stochastic collisions among themselves. If the energy deposition is larger than a critical value, the thermalized system becomes the QGP phase, whereas if it is lower, the system remains to be the hadron phase.

The features of the multi color-flux-tube system strongly depend on their density of the flux-tubes created by hard process in early stage. When the flux-tube number density is low enough, the system is approximated as the incoherent sum of the individual flux-tube. Its evolution is expected to be superposition of random multiple hadron creations of many color-flux-tubes produced between many nucleon-nucleon pairs. On the other hand, when the flux-tube number density is sufficiently high, many flux-tubes overlap each other and would be melted into a big flux-tube. During this process, each flux-tube loses its individuality and the whole system can be regarded as a huge flux-tube between heavy-ions like a condenser [4].

In this paper, we would like to study the properties of the multi flux-tube system. QCD is very hard to deal with in the infrared region analytically due to the breakdown of the perturbation technique. Moreover, for such a large scale system there is a severe limit on the computational power even in the lattice QCD simulation. Hence, we resort to use of the dual Ginzburg-Landau (DGL) theory [5,6] for this subject. This is the effective theory of nonperturbative QCD and can describe the color confinement, where there appears a long range linear potential between a quark-antiquark pair [7]. It is also able to describe the color-flux-tube as the dual version of the Abrikosov vortex [8].

In Sect.2 we formulate the system of multi color-flux-tubes by regarding it as that of two color-flux-tubes penetrating on a two dimensional sphere surface for neighboring flux-tubes. In this paper we shall study the qualitative aspects of the multi flux-tube system. We consider two cases; i.e. the directions of the color-flux-tubes are the same or alternative. The numerical results are discussed in Sect.3. Sect.4 is devoted to the concluding remarks and discussions on the QGP formation.

II. MULTI COLOR-FLUX-TUBE SYSTEM IN THE DUAL GINZBURG-LANDAU THEORY

In this section, we formulate the multi-color-flux system in the dual Ginzburg-Landau (DGL) theory. As 't Hooft proposed in 1981, SU(N_c) gauge theory is reduced to U$(1)^{N-1}$ gauge theory with the monopole by abelian gauge fixing [9]. In this gauge, there appears a monopole as a topological object, whose condensation leads to color confinement through the dual Meissner effect. Based on this idea, the DGL theory [5,6,10,11] was proposed as an effective theory of the nonperturbative QCD. The DGL lagrangian in the pure gauge system is written by using the dual gauge field, $B_\mu = \vec{B}_\mu \cdot \vec{H} = B_\mu^3 T^3 + B_\mu^8 T^8$, and QCD-monopole field, $\chi = \sqrt{2}\sum_a \chi_a E_a$ with $E_1 = \frac{1}{\sqrt{2}}(T_6 + iT_7)$, $E_2 = \frac{1}{\sqrt{2}}(T_4 - iT_5)$ and $E_3 = \frac{1}{\sqrt{2}}(T_1 + iT_2)$;

$$\mathcal{L}_{DGL} = \mathrm{tr}\hat{\mathcal{L}}$$

$$\hat{\mathcal{L}} = -\frac{1}{2}(\partial_\mu B_\nu - \partial_\nu B_\mu)^2 + [\hat{D}_\mu, \chi]^\dagger [\hat{D}_\mu, \chi] - \lambda(\chi^\dagger \chi - v^2)^2, \tag{1}$$

where $\hat{D}_\mu = \hat{\partial}_\mu + igB_\mu$ is the dual covariant derivative. The second term leads the usual form as following,

$$\begin{aligned}
\mathrm{tr}([\hat{D}_\mu, \chi]^\dagger [\hat{D}_\mu, \chi]) &= \mathrm{tr}([\hat{\partial}_\mu + igB_\mu, \chi^\dagger][\hat{\partial}_\mu + igB_\mu, \chi]) \\
&= 2\mathrm{tr}\{(\partial_\mu \chi_a^* E_{-a} + ig\vec{B}_\mu \chi_a^* [\vec{H}, E_{-a}])(\partial_\mu \chi_b E_b + ig\vec{B}_\mu \chi_b [\vec{H}, E_b])\} \\
&= \left|(\partial_\mu + ig\vec{\alpha}_a \cdot \vec{B}_\mu)\chi_a\right|^2,
\end{aligned} \tag{2}$$

where $\alpha_a (a = 1, 2, 3)$ are the root vectors of the SU(3) algebra, $\vec{\alpha}_1 = (-\frac{1}{2}, \frac{\sqrt{3}}{2})$, $\vec{\alpha}_2 = (-\frac{1}{2}, -\frac{\sqrt{3}}{2})$, $\vec{\alpha}_3 = (1, 0)$. The dual gauge field B_μ is defined on the dual gauge manifold $\mathrm{U}(1)_3^m \times \mathrm{U}(1)_8^m$, which is the dual space of the maximal torus subgroup $\mathrm{U}(1)_3^e \times \mathrm{U}(1)_8^e$ embedded in the original gauge group SU(3). The abelian field strength tensor is written as $F_{\mu\nu} = {}^*(\partial \wedge B)_{\mu\nu}$ so that the role of the electric and the magnetic field are interchanged in comparison with the ordinary A_μ description. The QCD-monopole has magnetic charge $g\vec{\alpha}_a$, where g is the dual gauge coupling constant. There appears the Dirac quantization condition, $eg = 4\pi$, with the electric charge gauge coupling constant e.

We consider the flux-tube with a quark and an antiquark at the both ends. In the standard notation [12,6], the quark charges are $\vec{Q}_a \equiv e\vec{w}_a$, where $\vec{w}_a (a = 1, 2, 3)$ are the weight vectors of the SU(3) algebra, $\vec{w}_1 = (\frac{1}{2}, \frac{1}{2\sqrt{3}})$, $\vec{w}_2 = (-\frac{1}{2}, \frac{1}{2\sqrt{3}})$, $\vec{w}_3 = (0, -\frac{1}{\sqrt{3}})$, for the three color states [12], red(R), blue(B) and green(G), respectively. Using the Gauss law, one finds the color electric field $\vec{\mathbf{E}}$ and then the dual gauge field $\vec{B}_\mu$, obeying $\vec{F}_{\mu\nu} = {}^*(\partial \wedge \vec{B})_{\mu\nu}$, are proportional to the color charge $\vec{Q}$. The QCD-monopole χ_a couples with the quark charge $\vec{Q}_b$ in the form of $\vec{\alpha}_a \cdot \vec{Q}_b \chi_a$. We note the algebraic relation between the root vector and the weight vector as,

$$2\vec{\alpha}_a \cdot \vec{w}_b = \sum_{c=1}^{3} \varepsilon_{abc} \in \{-1, 0, 1\}, \tag{3}$$

and therefore one kind of QCD-monopole χ_a couples with not three but two of the quark charges. For the example, in the case of (R-$\bar{R}$) system, $|\chi_1|$ is never affected; $|\chi_1| = v$. For this case, we can rewrite Eq.(1) as,

$$\mathcal{L}_{DGL} = -\frac{1}{3}\cdot\frac{1}{4}(\partial_\mu B^{\rm R}_\nu - \partial_\nu B^{\rm R}_\mu)^2 + 2\left|(\partial_\mu + \frac{i}{2}gB^{\rm R}_\mu)\chi^{\rm R}\right|^2 - 2\lambda(\left|\chi^{\rm R}\right|^2 - v^2)^2, \tag{4}$$

where $\vec{B}_\mu \equiv \vec{w}_1 B^{\rm R}_\mu$ and $|\chi_1| = v$, $\chi_2 = \chi^{\rm R*}$ and $\chi_3 = \chi^{\rm R}$, because the system is invariant under the transformation, $\chi_2^{\rm R*} \leftrightarrow \chi_3^{\rm R}$. In this case, we can rewrite the DGL lagrangian in the simple GL form;

$$\mathcal{L}_{DGL} = -\frac{1}{4}(\partial_\mu \hat{B}_\nu - \partial_\nu \hat{B}_\mu)^2 + \left|(\partial_\mu + i\hat{g}\hat{B}_\mu)\hat{\chi}\right|^2 - \hat{\lambda}(|\hat{\chi}|^2 - \hat{v}^2)^2, \tag{5}$$

where the field variables and coupling constants are redefined as

$$\hat{B}_\mu \equiv \frac{1}{\sqrt{3}}B^{\rm R}_\mu, \quad \hat{g} \equiv \frac{\sqrt{3}}{2}g, \quad \hat{\chi} \equiv \sqrt{2}\chi^{\rm R}, \quad \hat{v} \equiv \sqrt{2}v, \quad \hat{\lambda} \equiv \frac{\lambda}{2}. \tag{6}$$

We get the same expression for the other two cases (B-$\bar{B}$, G-$\bar{G}$). Hereafter, we will drop the notation $\hat{}$ since there is no confusion. The field equations for B_μ and χ are derived by the extreme condition,

$$\partial^2\chi + 2igB^\mu(\partial_\mu\chi) + ig(\partial_\mu B^\mu)\chi - g^2 B_\mu^2\chi + 2\lambda(|\chi|^2 - v^2)^2\chi = 0, \tag{7}$$

$$\partial_\mu(\partial^\mu B^\nu - \partial^\nu B^\mu) + ig\{(\partial^\nu\chi^*)\chi - (\partial^\nu\chi)\chi^*\} + 2g^2 B^\nu|\chi|^2 = 0. \tag{8}$$

We consider two ideal cases of multi color-flux-tube penetrating on a *two dimensional plane*. The directions of all the color-flux-tubes are the same[Fig.2a] in one case or alternative[Fig.2b] in the other case. When the flux-tubes are long enough, the effect of the flux edges is negligible. Hence, taking the direction of the flux-tubes as the z-axis, the system is translationally invariant in the z-direction and is essentially described only with two spatial coordinates (x, y). For the periodic case in the (x, y) coordinate, we can regard the system as two flux-tubes going through two poles (north and south poles) of the S^2 *sphere*. For the system of flux-tubes with all the directions being the same, we take two flux-tubes passing through the two poles on S^2 sphere as shown in Fig.2(a) (which we call the two flux-tubes system). For the alternative case, on the other hand, we take a flux-tube coming in from the south pole and the other leaving out through the north pole (which we call flux-tube

and anti-flux-tube system). Such a prescription leads the exact solution for the periodic crystal of the sine-Gordon kinks, and also provides a simple but good description for the finite density Skyrmion system studied by Manton [13,14].

The two color-flux-tube system on the sphere S^2 with radius R corresponds to the multi-flux-tube system with the density $\rho = 1/(2\pi R^2)$. Introducing the polar coordinates (R, θ, φ) on S^2, we consider the static solution satisfying

$$B_0 = 0, \qquad \mathbf{B} = B(\theta)\mathbf{e}_\varphi \equiv \frac{\tilde{B}(\theta)}{R\sin\theta}\mathbf{e}_\varphi, \qquad \chi = \bar{\chi}(\theta)e^{in\varphi}, \tag{9}$$

where we have used the axial symmetry of the system. Here the electric field penetrates vertically on the *sphere* surface, $\mathbf{E}//\mathbf{e}_r$;

$$\mathbf{E} = \nabla \times \mathbf{B} = E\mathbf{e}_r, \tag{10}$$

which corresponds to the electric field penetrating vertically on the *plane*, $\mathbf{E}//\mathbf{e}_z$. The field equations are given by

$$\frac{1}{R^2\sin\theta}\frac{d}{d\theta}(\sin\theta\frac{d\bar{\chi}}{d\theta}) - [\frac{1}{R^2\sin^2\theta}(n - g\tilde{B}(\theta))^2 + 2\lambda(\bar{\chi}^2 - v^2)]\bar{\chi} = 0, \tag{11}$$

$$\frac{d}{R^2 d\theta}(\frac{1}{\sin\theta}\frac{d}{d\theta}\tilde{B}(\theta)) + \frac{2g}{\sin\theta}(n - g\tilde{B}(\theta))\bar{\chi}^2 = 0. \tag{12}$$

Consider the closed loop C on S^2 with a constant polar angle $\theta = \alpha$ and $\phi \in [0, 2\pi)$, the electric flux penetrating the area surrounded by the loop C is given by

$$\Phi(\alpha) = \int_S \mathbf{E}\cdot d\mathbf{S} = \int \nabla\times\mathbf{B}\cdot d\mathbf{S} = \oint_c \mathbf{B}\cdot d\mathbf{l} = 2\pi\tilde{B}(\alpha). \tag{13}$$

The boundary conditions for the two flux-tubes system as shown in Fig.2(a) are given as

$$\Phi(\alpha) = 2\pi\tilde{B}(\alpha) = 0, \qquad \bar{\chi}(\alpha) = 0 \qquad \text{as} \quad \alpha \to 0, \tag{14}$$

$$\Phi(\alpha) = 2\pi\tilde{B}(\alpha) = \frac{1}{2}\int_S ER^2 d\Omega = \pm\frac{2\pi n}{g} \qquad \text{as} \quad \alpha \to \frac{\pi}{2} \pm \epsilon, \tag{15}$$

Here, n corresponds to the topological number of the flux-tube, which appears also in the vortex solution in the superconductivity. This boundary condition at $\alpha \to \frac{\pi}{2} \pm \epsilon$ has a

discontinuity for the dual gauge field $\tilde{B}$. Because the electric flux leaves out from the two poles, there should be some sources to provide the electric flux. In this case, the Dirac-string like singularity appears on the $\theta = \pi/2$ line, through which the electric flux comes into the sphere from long distance. For the flux-tube and anti-flux-tube system as shown in Fig.2(b), the boundary condition around $\theta = 0, \pi$ is given as

$$\Phi(\alpha) = 2\pi\tilde{B}(\alpha) = 0, \qquad \bar{\chi}(\alpha) = 0 \qquad \text{as} \quad \alpha \to 0, \pi. \tag{16}$$

In this case, there does not appear the Dirac-string like singularity, since the electric flux is conserved. The free energy for the unit length of the color-flux-tube is written as

$$\begin{aligned} F &= \int R^2 d\Omega \left[\frac{1}{2}\left(\frac{1}{R^2\sin\theta}\frac{d}{d\theta}\tilde{B}(\theta)\right)^2 + \left(\frac{1}{R}\frac{d\bar{\chi}}{d\theta}\right)^2\right] \\ &+ \int R^2 d\Omega \left[\frac{1}{R^2\sin^2\theta}(n - g\tilde{B}(\theta))^2\bar{\chi}^2 + \lambda\left(\bar{\chi}^2 - v^2\right)^2\right] \\ &= \int_0^{\theta=\pi} 2\pi\,\sin\theta d\theta \left[\left(\frac{d\bar{\chi}}{d\theta}\right)^2 + \frac{1}{\sin^2\theta}(n - g\tilde{B}(\theta))\bar{\chi}^2\right] \\ &+ \int_0^{\theta=\pi} 2\pi\,\sin\theta d\theta \left[\frac{1}{2}\left(\frac{1}{\sin\theta}\frac{d}{d\theta}\tilde{B}(\theta)\right)^2\right] \cdot \frac{1}{R^2} + \int_0^{\theta=\pi} 2\pi\,\sin\theta d\theta \left[\lambda\left(\bar{\chi}^2 - v^2\right)^2\right] \cdot R^2. \end{aligned} \tag{17}$$

First, we consider a limit of $R \to \infty$, which corresponds to the ordinary single vortex solution. Introducing a new variable $\rho \equiv R\sin\theta$, the free energy in the limit $R \gg \rho(\theta \sim 0)$ is written as

$$\begin{aligned} F &= \int 2\pi\rho d\rho \left[\frac{1}{2}\left(\frac{1}{\rho}\frac{d}{d\rho}\rho B(\rho)\right)^2 + \left(\frac{d\bar{\chi}}{d\rho}\right)^2\right] \\ &+ \int 2\pi\rho d\rho \left[\frac{1}{\rho^2}(n - g\rho B(\rho))\bar{\chi}^2 + \lambda\left(\bar{\chi}^2 - v^2\right)^2\right], \end{aligned} \tag{18}$$

and the field equations are

$$\frac{1}{\rho}\frac{d}{d\rho}\rho\left(\frac{d}{d\rho}\bar{\chi}\right) - \frac{1}{\rho^2}(n - g\rho B(\rho))^2 - 2\lambda\left(\bar{\chi}^2 - v^2\right)\bar{\chi} = 0, \tag{19}$$

$$\frac{d}{d\rho}\frac{1}{\rho}\frac{d}{d\rho}(\rho B(\rho)) + \frac{2g}{\rho}(n - g\rho B(\rho))\bar{\chi}^2 = 0, \tag{20}$$

with the boundary condition,

$$\Phi = 2\pi\rho B(\rho)|_0^\infty = \frac{2\pi n}{g} \quad \text{as} \quad \rho \to \infty. \tag{21}$$

Above equations, (18-20), coincide exactly with those of ordinary single vortex solution in the cylindrical coordinate. Thus, we get the desired results.

One can analytically investigate the dependence of the profile functions($\tilde{B}(\theta), \chi(\theta)$) on the flux-tube number density. For this purpose, we express the free energy as

$$F \equiv f_0 + f_E \cdot \frac{1}{R^2} + f_\chi \cdot R^2, \tag{22}$$

where f_0, f_E, andf_χ are R independent functions and written as

$$f_0 \equiv \int_0^{\theta=\pi} 2\pi \sin\theta d\theta \left[\left(\frac{d\bar{\chi}}{d\theta} \right)^2 + \frac{1}{\sin^2\theta} (n - g\tilde{B}(\theta))^2 \bar{\chi}^2 \right], \tag{23}$$

$$f_E \equiv \int_0^{\theta=\pi} 2\pi \sin\theta d\theta \frac{1}{2} \left(\frac{1}{\sin\theta} \frac{d}{d\theta} \tilde{B}(\theta) \right)^2, \tag{24}$$

$$f_\chi \equiv \int_0^{\theta=\pi} 2\pi \sin\theta d\theta \lambda \left(\bar{\chi}^2 - v^2 \right)^2. \tag{25}$$

In the large R case, which corresponds to the small color-flux-tube number density in the original multi-flux-tube system, the third term $f_\chi R^2$ is dominant. Hence, the free energy F is minimized as $\bar{\chi} \sim v$, that is, the QCD-monopole tends to condense, and then the color electric field is localized only around $\theta = 0$ (north pole) and $\theta = \pi$ (south pole) as shown in Fig.3. On the other hand, in the small R case, the second term f_E/R^2 is dominant. There is a constraint on the total flux penetrating on the upper sphere,

$$\Phi \equiv \int_0^{\frac{\pi}{2}} E(\theta) 2\pi R^2 \sin\theta d\theta = \frac{2n\pi}{g}, \tag{26}$$

that is,

$$\int_0^1 E(t)dt = \frac{n}{gR^2} \equiv C \quad \text{with} \quad t = \cos\theta. \tag{27}$$

Hence, one finds the equation,

$$f_E \propto \int_0^{\frac{\pi}{2}} E(\theta)^2 \sin\theta d\theta = \int_0^1 E(t)^2 dt = \int_0^1 \{(E(t) - C)^2\} dt + C^2 \geq C^2. \tag{28}$$

This condition leads to the uniform color electric field $E = C$, which provides the minimum of f_E. Thus, the color electric field tends to spread over the space uniformly.

Finally we consider the critical radius R_c of the phase transition to the normal phase, where the QCD-monopole disappears. There are three useful inequalities on f_E, f_χ, and F,

$$f_E \geq 2\pi(\frac{n}{g})^2 \tag{29}$$

$$0 \leq f_\chi \leq \ 4\pi\lambda v^4, \tag{30}$$

$$F = f_0 + f_E \cdot \frac{1}{R^2} + f_\chi \cdot R^2 \geq f_0 + 2\sqrt{f_E f_\chi}. \tag{31}$$

The equality is satisfied in Eq.(31),

$$R^4 = \frac{f_E}{f_\chi} \tag{32}$$

Using the inequalities equations(29-31), R^4 is larger than the critical R_c^4;

$$R^4 \geq R_c^4 \equiv \frac{2\pi n^2/g^2}{4\pi\lambda v^4} = (\frac{n^2}{2\lambda g^2})\frac{1}{v^4}. \tag{33}$$

For $R > R_c$, there exists a nontrivial inhomogeneous solution, which differs from the normal phase. For $R \leq R_c$, homogeneous normal phase provides the minimum of F. Thus, R_c is the critical radius of the phase transition from the flux-tube phase to the normal one. In this case, the critical radius and the electric field are given by

$$\rho_c = \frac{1}{2\pi R_c^2} = \sqrt{\frac{\lambda}{2}}\frac{gv^2}{\pi n}, \tag{34}$$

$$E_c = \frac{n}{gR_c^2} = \sqrt{2\lambda}v^2, \tag{35}$$

respectively.

III. NUMERICAL RESULTS

We start with showing the low density case of two flux tubes system in Fig.3. The parameters of the DGL theory are fixed as $\lambda = 25$, $v = 0.126$GeV, which lead the masses

$m_B = 0.5$GeV and $m_\chi = 1.26$GeV [6]. This parameter set provides the flux-tube radius $r_{FT} \sim$0.4fm and the suitable interquark potential with the string tension as $\sigma =$1GeV/fm. The QCD-monopole condensate $\bar{\chi}(\theta)$, the dual gauge field $\tilde{B}(\theta)$ and the color electric field $E(\theta)$ are plotted as functions of the polar angle θ. The electric field $E(\theta)$ is localized around the two poles(θ=0 and π) and drops suddenly as θ deviates from the two poles. The QCD-monopole condensate vanishes at the two poles and becomes constant in the region away from these poles. This behavior corresponds to the case of independent two vortices in superconductivity. Different from these physical quantities, the dual gauge field $\tilde{B}(\theta)$ is discontinuous at $\theta = \pi/2$. There should be Dirac-string like source to provide, because the electric flux leaving out from the two poles. It should be noted that the system has the reflection symmetry on $\theta = \frac{\pi}{2}$ plane.

We show now in Fig.4 the number density dependence of the flux-tubes. For the large radius of the sphere, ($R \geq$2fm), the color electric flux is localized at $\theta = 0$ and π and there the QCD-monopole condensate vanishes, while the $\bar{\chi}$ becomes constant $\bar{\chi} \simeq v$ around $\theta = \frac{\pi}{2}$. As the radius R decreases, the electric flux, localized at $\theta = 0$, $\theta = \pi$, starts to overlap, and the value of the QCD-monopole condensate $\bar{\chi}$ becomes small. The electric field $E(\theta)$ becomes constant and $\bar{\chi}$ vanishes below a critical radius R_c. We show in Fig.5 the QCD-monopole condensate at $\theta = \frac{\pi}{2}$ (the maximum value of the QCD-monopole condensate) as a function of the sphere radius R (flux-tube number density $\rho = 1/(2\pi R^2)$). The (first order) phase transition occurs and the system becomes homogeneous normal phase above the critical value of the flux-tube number density.

Here, we compare the free energy of two flux tube system with that of inhomogeneous system in Fig.6. At large R, the former is smaller and the system favors the existence of two flux tubes. As R becomes smaller, the energy difference of the system becomes smaller. Two flux-tubes melt and the electric field are changed to be homogeneous below the critical radius $R_c = 0.35$fm, which corresponds to the critical density $\rho_c = 1/(2\pi R_c^2) = 1.3\text{fm}^{-2}$. This critical density agrees with the analytic estimation in Eq.(34), $\rho_c = \sqrt{\frac{\lambda}{2}}\frac{gv^2}{\pi n} = 1.3\text{fm}^{-2}$.

We discuss now the system of flux-tube and anti-flux-tube with opposite direction placed

at θ =0 and $\theta = \pi$ respectively as shown in Fig.7. At low flux-tube number density ($R \geq 2$fm), the flux-tube is localized at $\theta = 0$, while the anti-flux-tube at $\theta = \pi$. The QCD-monopole condensate $\bar{\chi}(\theta)$ vanishes at the two poles ($\theta = 0, \pi$) and becomes constant away from these poles. As R decreases, the electric flux starts to cancel each other and the QCD-monopole condensate becomes small. We also compare the free-energy of the flux-tube and anti-flux-tube system with the homogeneous system, in which both QCD-monopole condensate and electric field are vanished. The critical radius, $R_c = 0.31$fm, is similar to the value of the two flux-tubes system.

IV. SUMMARY AND CONCLUDING REMARKS

We have studied the system of multi color-flux-tubes using the dual Ginzburg-Landau (DGL) theory. The DGL theory provides the color-flux-tube between a quark and an anti-quark pair as the dual version of the Abrikosov vortex. We have considered two ideal cases, where the directions of all the color-flux-tubes are the same and alternative penetrating on a two-dimensional plane. In order to treat these cases in a simple way, we have introduced color-flux-tubes on a sphere S^2 going through its north and south poles. Here, the flux-tube system on the sphere S^2 with the radius R corresponds to the multi-flux-tube system with the flux-tube number density $\rho = 1/(2\pi R^2)$. The energy minimization condition of the system provides a simple coupled diffential equation for the dual gauge field $\tilde{B}(\theta)$ and QCD-monopole condensate $\bar{\chi}(\theta)$ as functions of the polar angle θ. We have solved the coupled equation numerically with the parameter set, which provides the radius of flux tube as $r_{FT} = 0.4$fm in the flat space ($R \to \infty$).

We have found in both cases that the solution behaves as two independent flux-tubes at a large R, i.e., a small number density ρ. As the radius R decreases, the QCD-monopole condensate decreases and eventually vanishes at a critical density, where the color-electric field E becomes uniform. The critical density ρ_c is found as ρ_c =1.3fm^{-2} = (1.14fm)$^{-2}$ for the two color-flux tube system; ρ_c =1.7fm^{-2} = (1.3fm)$^{-2}$ for the flux-tube and anti-flux-

tube system. Such similar values in both cases suggest that an actual flux-tube system would become uniform around similar density to $\rho_c \sim$1.5fm^{-2}, because realistic flux-tube system includes the flux-tubes and anti-flux tubes randomly, which would correspond to an intermediate system between the above two ideal cases. Thus, the configuration of the color-electric field and the QCD-monopole field depend largely on the flux-tube density.

As discussed above, many flux-tubes are melted around $\rho_c \sim 1.5$ fm^{-2}. Let us discuss here in case of central collision between heavy-ions with mass number A. The nuclear radius R is given by $R = R_0 A^{1/3}$, where R_0=1.2fm corresponds to the nuclear radius, and the normal baryon-number density is $\frac{A}{\frac{4}{3}\pi R^3} = \frac{1}{\frac{4}{3}\pi R_0^3}$. In the central region, one nucleon in the projectile makes hard collisions with $\frac{3}{2}A^{1/3}(= 2\pi R_0^3 A^{1/3} \times \rho_0)$ nucleons in the target, because the reaction volume is $\pi R_0^2 \times 2R = 2\pi R_0^3 A^{1/3}$. Hence, nucleon-nucleon collision number is expected as $(\frac{3}{2}A^{1/3})^2 = \frac{9}{4}A^{2/3}$ per the single-nucleon area πR_0^2 between projectile and target nuclei. Assuming one flux-tube formed in one nucleon-nucleon hard collision, the flux-tube density is estimated as $\rho = \frac{\frac{9}{4}A^{2/3}}{\pi R_0^2} = \frac{A^{2/3}}{(1.4\mathrm{fm})^2}$. For instance, ρ would be 4.5, 5.8 and 17.5 fm^{-2} for $A = 27$, 40 and 208. This would indicate that the scenario of creating large sizes of flux tube becomes much relevant for A-A collision with larger A.

It would be important to reconsider the process of the QGP formation in terms of the flux-tube number density. When the flux-tube density is low enough, the flux-tubes are localized. Each flux-tube evolution would be regarded as the multi creation of hadrons in the high energy p-p collision via the flux-tube breaking. In this process, q-$\bar{q}$ pair creation plays an essential role on the QGP formation, which is the usual scenario.

On the other hand, for the dense flux-tube system, neighboring flux-tubes are melted into a large cylindrical tube, where QCD-monopole condensate disappears. Such a system, where the color electric field is made between heavy-ions, becomes approximately homogeneous and is regarded as the 'color condenser'. In this case, large homogeneous QGP may be created in the central region.

In the actual case, however, the variations and directions of the color-flux-tubes are random. For instance, in the peripheral region, flux-tubes would be localized and are broken by

quark-antiquark pair creations. In the central region, dense flux-tubes are melted by annihilation or unification [15] of flux-tubes. In this region, a huge system of dynamical gluons appears, because many dynamical gluons are created during this process. Thermalization of such quarks and gluons leads to QGP. Thus, the process of QGP formation should depend largely on the density of created flux-tubes, which is closely related to the incident energy, the impact parameter and the size of the projectile and the target nuclei.

Figure caption

Fig.1 The scenario of the QGP formation in high energy heavy ion collisions. (a) There appear many color-flux-tubes between the projectile nucleus and the target nucleus just after the collisions. (b) When the distance between the two nuclei becomes large, there appears the pair creation of quark and anti-quark. (c) Many created quarks and anti-quarks make frequent collisions to form a thermal equilibrium and form QGP when the energy density is larger than the critical value.

Fig.2 (a) A multi color-flux system with the same direction of the flux-tubes is approximated by the two color-flux-tubes going out from the north and the south poles on a sphere S^2. (b) A multi color-flux system, where flux-tube direction is alternative, is approximated by the flux-tube and 'anti-flux-tube' system penetrating on S^2 with the color-flux going in from the south pole and leaving out from the north pole.

Fig.3 The color-electric field $E(\theta)$(solid curve), the QCD-monopole condensate $\bar{\chi}(\theta)$(dashed curve) and the dual gauge field $B(\theta)$ (thin solid curve) are plotted as functions of the polar angle θ for the low density case; R =4.0fm.

Fig.4 We show the case of the two flux-tubes system on penetrating S^2. The color-flux $E(\theta)$ and the QCD-monopole condensate $\bar{\chi}(\theta)$ are depicted as functions of the polar angle θ for the three radii; R =2.0fm, 0.5fm and 0.347fm. Below the critical radius R_c=0.347fm, the color electric field E becomes constant and the QCD-monopole condensate $\bar{\chi}$ vanishes entirely.

Fig.5 We show the R (flux-tube density, $1/(2\pi R^2)$) dependence of QCD-monopole configuration. The QCD-monopole condensate at $\theta = \frac{\pi}{2}$ decreases, as the radius R becomes smaller, and vanishes below $R = R_c$.

Fig.6 We show the free energy in two color-flux-tubes system (solid curve) and uniform system (dashed curve). Because the lower free energy system is realized, the inhomogeneous system is changed into homogeneous system below the critical radius, $R_c = 0.35$fm ($\rho_c = 1/(2\pi R_c^2) = 1.3\text{fm}^{-2}$).

Fig.7 We show the case of the flux the and anti-flux-tube penetrating on S^2 system. The

color-electric field $E(\theta)$ and the QCD-monopole condensate $\bar{\chi}(\theta)$ are depicted as functions of the polar angle θ for the three radii; R =2.0fm, 0.5fm, and 0.315fm. Below R =0.315fm, both the color electric flux and the QCD-monopole condensate vanish entirely.

REFERENCES

[1] G. Boyd, J. Engels, F. Karsch, E. Laermann, C. Legeland, M. Lutgemeier and B. Petersson, Phys. Rev. Lett. **75** (1995) 4169.
J. Fingberg, F. Karsch and U. M. Heller, Nucl. Phys. (Proc. Suppl.) **B30** (1993) 343.

[2] K. Kanaya, Prog. Theor. Phys. (Suppl.) **120** (1995) 25.
Y. Iwasaki, K. Kanaya, S. Sakai, T. Yoshie, Nucl. Phys. (Proc. Suppl.) **B26** (1992) 311.

[3] J. D. Bjorken, Phys. Rev. **D27** (1983) 140.

[4] N. K. Glendenning and T. Matsui, Phys. Rev. **D28** (1983) 2890;
Phys. Lett. **B141** (1984) 419.

[5] T. Suzuki, Prog. Theor. Phys. **80** (1988) 929.
S. Maedan and T. Suzuki, Prog. Theor. Phys. **81** (1989) 229; **81** (1989) 752.

[6] H. Suganuma, S. Sasaki and H. Toki, Nucl. Phys. **B435** (1995) 207.
H. Suganuma, H. Ichie, S. Sasaki and H. Toki, Proc. of Int. Workshop on "Color Confinement and Hadrons", (World Scientific, 1995) 65.

[7] H. Suganuma and T. Tatsumi, Phys. Lett. **B269** (1991) 371.
H. Suganuma and T. Tatsumi, Prog. Theor. Phys. **90** (1993) 379.

[8] H. B. Nielsen and P. Olesen, Nucl. Phys. **B61** (1973) 45.

[9] G. 't Hooft, Nucl. Phys. **B190** (1981) 455.

[10] Z. F. Ezawa and A. Iwazaki, Phys. Rev. **D25** (1982) 2681.
Z. F. Ezawa and A. Iwazaki, Phys. Rev. **D26** (1982) 631.

[11] H. Ichie, H. Suganuma and H. Toki, Phys. Rev. **D52** (1995) 2944 .
H. Ichie, H. Monden, H. Suganuma and H. Toki, Proc. of Int. Symp. on "Nuclear Reaction Dynamics of Nucleon-Hadron Manybody System", Dec. 1995, RCNP Osaka.

[12] K. Huang, "Quarks Leptons and Gauge Field", (World Scientific, 1982) 1.

[13] T. D. Lee, "Particle Physics and Introduction to Field Theory" (Harwood Academic Publishers, 1926) 1.

[14] N. S. Manton, Commun. Math. Phys. **111** (1987) 469.

[15] H. Suganuma, S. Sasaki, H. Toki and H. Ichie, Prog. Theor. Phys. (Suppl.) **120** (1995) 57.